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Synchrotron Radiation and its Comptonization in Astrophysical Thermal Plasmas

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Abstract. We investigate the process of Comptonization of synchrotron radiation in thermal plasma of the optical depth $\tau_{\rm es} \sim 1$. Approximate expressions for the synchrotron emission coefficients are derived. We calculate the Comptonization spectrum and present an analytical expression for the intensity of the Comptonized synchrotron radiation.

1. Introduction

It is widely accepted that the inverse Compton effect is responsible for the power-law spectrum in X- and γ-rays observed both in galactic black hole candidates (GBHCs) and in AGN. Typically the observed spectra can be well modeled by Comptonization in thermal plasma of the electron temperature $\sim 10^9$ K and Thomson optical depth $\tau_{\rm es} \sim 1$. A remaining question, strongly related to accretion disc models, is the nature of the source of soft photons undergoing Comptonization. It is often suggested that synchrotron radiation can play the role both in hot, optically thin discs (Narayan & Yi 1996) and in cold discs with coronae (Di Matteo et al. 1997). However it is necessary to describe Comptonization of synchrotron radiation more precisely than before in order to calculate the energy balance properly and obtain reliable disc models. Therefore in this work we consider synchrotron emission from thermal plasma (section 2), obtain approximate formulae for the emission coefficients and then (section 3) calculate the Comptonization spectrum, deriving analytical formulae for the intensity of the radiation.

2. Synchrotron Radiation from Thermal Plasma

When an electron spirals around magnetic field lines it emits radiation and the Fourier analysis allows one to find its spectrum (see e.g. Pacholczyk, 1970 for detailed discussion). The results are as follows: consider an electron (of charge e) moving in uniform magnetic field, $\mathbf{B}$, at a velocity $\beta \equiv \mathbf{v}/c$. Let ξ be the angle between $\mathbf{v}$ and $\mathbf{B}$, $\mu = \cos\xi$, and let ϑ be the angle between $\mathbf{B}$ and the direction towards the observer. Then the synchrotron emission, as measured by the observer (expressed e.g. in erg sr^{-1}; throughout our work we use Gauss units) is given by

$$\frac{dW}{d\nu\, d\Omega\, dt}(\vartheta, \mu, \gamma) = \frac{2\pi e^2 \nu^2}{c} \times$$

$$\sum_{m=1}^{\infty} \delta(y_m) \left[\left(\frac{\cos\vartheta - \beta\mu}{\sin\vartheta} \right)^2 J_m^2(x) + (1-\mu^2)\beta^2 J_m'^2(x) \right], \tag{1}$$

where

$$y_m \equiv \frac{m\nu_{\rm c}}{\gamma} - \nu(1-\beta\mu\cos\vartheta), \quad x \equiv \frac{\nu\gamma\beta\sin\vartheta(1-\mu^2)^{1/2}}{\nu_{\rm c}}, \tag{2}$$

$\nu_{\rm c} \equiv eB/2\pi m_{\rm e}c$ is the cyclotron frequency, $\gamma = (1-\beta^2)^{-1/2}$ is the Lorentz factor, and J_m is the Bessel function of order m.

Equation (1) shows that the spectrum consists of discrete harmonics at the frequencies, $\nu_m = m\nu_{\rm c}/(\gamma(1-\beta\mu\cos\vartheta))$. The relative separation between harmonics decreases with m, and thus the above discrete synchrotron spectrum can be approximated by a continuum at large m.

The emission coefficient $j_\nu(\vartheta)$ of thermal plasma with temperature T remaining in uniform magnetic field, $\mathbf{B}$, can be obtained by integrating the expression (1) over a relativistic Maxwellian electron distribution,

$$f(\gamma,\mu) = \frac{n_{\rm e}}{2\Theta} \frac{\gamma(\gamma^2-1)^{1/2}}{K_2(1/\Theta)} \exp\left(-\frac{\gamma}{\Theta}\right). \tag{3}$$

Here $\Theta \equiv kT/m_{\rm e}c^2$ is the dimensionless plasma temperature, $m_{\rm e}$ and $n_{\rm e}$ are the electron mass and number density, respectively, and K_2 is the modified Bessel function. The integral for the emission coefficient is then

$$j_\nu(\vartheta) = \int_1^\infty {\rm d}\gamma \int_{-1}^{1} {\rm d}\mu\, \eta_\nu(\vartheta,\mu,\gamma) f(\gamma,\mu). \tag{4}$$

Petrosian (1981, hereafter P81) presented an elegant solution of the problem of integrating (4) by replacing the summation over m in equation (1) by an integration over ν, employing an analytical approximation of J_m [eq. (9.3.7) of Abramowitz & Stegun (1970)], and then integrating using the method of the steepest descent.

The method of integration of P81 yields, with some additional approximations,

$$j_\nu(\vartheta) = \frac{2^{1/2}\pi e^2 n_{\rm e}\nu}{3c\,K_2(1/\Theta)} \exp\left[-\left(\frac{9\nu}{2\nu_{\rm c}\Theta^2 \sin\vartheta} \right)^{1/3} \right]. \tag{5}$$

We see that the emission coefficient becomes very small both at large frequencies and at viewing angles ϑ significantly different from $\pi/2$.

We checked the accuracy of this formula, compared to the results of numerical integration of (4), in the range $50 \le \nu/\nu_{\rm c} \le 1000$ and $0.05 \le \Theta \le 3$. For small temperatures (5) strongly overestimates the true value of j_ν and is therefore useless for $\Theta < 0.1$, while for $0.1 \lesssim \Theta \lesssim 0.2$ one can obtain results accurate to the order of magnitude or better. For temperatures $0.2 < \Theta \lesssim 1$ the error generally does not exceed 30%. For temperatures higher than $\Theta = 1$ the errors become larger again, but even for $\Theta = 3$ one can still obtain estimations correct to the order of magnitude. The errors also diminish with increasing frequency.

We note that the corresponding formula of equation (26) in P81 appears too small by a factor of 2 due to the adopted normalization of the electron distribution $f(\gamma,\mu)$ [equation (23) in P81] being also twice too small. On the other hand the same formula cited in expression (26) of Mahadevan, Narayan & Yi (1996, hereafter M96) is too large by a factor 4π compared to the correct result. Takahara & Tsuruta (1982) obtain a formula identical to our equation (5) at $\vartheta=\pi/2$. We have also checked that the emission coefficient computed by Jones & Hardee (1979) for relativistic plasma, i.e. $\Theta \gg 1$, is larger than the corresponding limit of equation (5) by a factor $\sqrt{2}$.

We see that that the accuracy of the emission coefficient (5) is not satisfactory at lower temperatures, as also found by M96. This can be significantly improved using the P81 formula (11) with the approximations from P81 expression (25), which yields an emission coefficient more accurate than that of equation (5),

$$j_\nu(\vartheta)=\frac{\pi e^2}{c}\left(\frac{\nu\nu_{\rm c}\Theta}{6\gamma_0}\right)^{1/2} f(\gamma_0)\left(1+2\frac{\cot^2\vartheta}{\gamma_0^2}\right)\left(1-\beta_0^2\cos^2\vartheta\right)^{1/4}\mathcal{Z}(\gamma_0). \qquad (6)$$

Here $\mathcal{Z}$ corresponds to $\mathcal{Z}_{\rm max}^{2m}$ in the notation of P81,

$$\mathcal{Z}(\gamma)\equiv\left[\frac{t\exp\left[(1+t^2)^{-1/2}\right]}{1+(1+t^2)^{1/2}}\right]^{2m}, \quad t\equiv\beta\gamma\sin\vartheta, \quad m\equiv\frac{\nu(1+t^2)}{\nu_{\rm c}\gamma}, \qquad (7)$$

$f(\gamma)=2f(\gamma,\mu)$ due to isotropy of the electron distribution, and γ_0 is the Lorentz factor corresponding to the saddle point in the integral (4) over γ,

$$\gamma_0=\left(\frac{4\nu\Theta}{3\nu_{\rm c}\sin\vartheta}\right)^{1/3}, \quad \beta_0=\sqrt{1-\frac{1}{\gamma_0^2}}. \qquad (8)$$

In fact γ_0, as a function of ν, represents the energy of the electrons that contribute most to the radiation at a given frequency ν.

Even for $\Theta=0.05$ and $\nu/\nu_{\rm c}=50$ the formula (6) gives an order of magnitude accuracy or better (again the accuracy increasing with the frequency), while for $0.2\lesssim\Theta\lesssim 0.5$ the error does not exceed 30%. For higher temperatures, $0.5<\Theta\lesssim 1$, the 30% accuracy can be achieved only for $\nu/\nu_{\rm c}>100$. For $\Theta>1$ the accuracy of (6) is comparable to (5).

The accuracy of formula (6) for lower temperatures can be further improved by calculating the saddle point γ_0 numerically as the maximum of the function $g(\gamma)=f(\gamma)/\gamma\mathcal{Z}(\gamma)$, instead of using γ_0 of equation (8). With this modification, for $0.05\lesssim\Theta\lesssim 0.2$ the accuracy of (6) is always better than 20%.

The emission coefficient derived above is appropriate for plasma in uniform magnetic field. However, in typical astrophysical conditions we often expect either to deal with emission from regions where magnetic fields are chaotic or to observe radiation being a sum of contributions from a number of regions with different orientations of magnetic field. Therefore it is important to find the emission coefficient averaged over magnetic field directions, or, equivalently, over observation angles,

$$\bar{j}_\nu=\frac{1}{4\pi}\int j_\nu(\vartheta){\rm d}\Omega=\frac{1}{2}\int_0^\pi j_\nu(\vartheta)\sin\vartheta\,{\rm d}\vartheta. \qquad (9)$$

We have obtained an expression for $\bar{j}_\nu$ by integrating equation (6) over ϑ with the method of the steepest descent. Although this method turns out not to give accurate results in the present case, we have found that the main discrepancy with results of exact integration in a wide range of parameters is the normalization remaining too large by a factor of ≈ 1.71. Therefore the resulting approximated formula reads

$$\bar{j}_\nu = \frac{1}{1.71} \frac{\pi^{3/2} e^2}{2^2 3^{1/2} c} f(\gamma_0) \left(\frac{\nu \nu_{\rm c} \Theta}{\gamma_0} \right)^{1/2} \mathcal{Z}(\gamma_0) \left| \frac{{\rm d}^2 \ln \mathcal{Z}(\gamma_0)}{{\rm d}\vartheta^2} \right|_{\vartheta=\pi/2}^{-1/2}, \tag{10}$$

where γ_0 is given by equation (8). We have checked that in the range $0.1 \le \Theta \le 2$ and $50 \le \nu/\nu_{\rm c} \le 1000$ the additional error, introduced by this approximate integration is always smaller than 10%. This error again diminishes with increasing frequency.

On the other hand, the asymptotic emission coefficient (5) can be integrated over ϑ, as done by M96. This yields

$$\bar{j}_\nu = \frac{2^{1/6} \pi^{3/2} e^2 n_{\rm e} \nu}{3^{5/6} c K_2(1/\Theta) y^{1/6}} A(\Theta, y) \exp \left[- \left(\frac{9y}{2} \right)^{1/3} \right], \tag{11}$$

where $y \equiv \nu/\nu_{\rm c}\Theta^2$ and the correction factor A represents the ratio of the exact emission coefficient to that obtained by integration of equation (5). This factor can be calculated from the ratio of equation (33) to (31) in M96 with fitting coefficients of their Table 1. In the range $0.16 \le \Theta \lesssim 2.7$ and $100 \lesssim \nu/\nu_{\rm c} \lesssim 1000$ the results of formula (10) are by no more than 30% smaller than (11), the latter calculated with the tabulated corrections. The results of (10) and (11) get closer as the frequency grows.

The synchrotron radiation is self-absorbed by electrons up to some critical frequency (the *turnover frequency*) $\nu_{\rm t}$, above which plasma becomes optically thin to the synchrotron radiation, i.e.

$$\tau = \alpha_{\nu_{\rm t}} R = 1 \tag{12}$$

where R is the characteristic size of the plasma. Below $\nu_{\rm t}$, the observed emission has the blackbody, typically Rayleigh-Jeans, spectrum. Equation (12) with $\alpha_{\nu_{\rm t}}$ calculated from Kirchoff's law can be solved for $\nu_{\rm t}$ numerically. For an approximate, analytical expression see Zdziarski et al. (1998).

3. Comptonization of Synchrotron Photons

Synchrotron photons undergoing Comptonization in a medium with Thomson optical depth $\tau_{\rm es} \sim 1$ can be well approximated as monochromatic, at the frequency $\nu_{\rm t}$. This follows from the fact that both the absorption and emission coefficients of synchrotron radiation fall steeply in the vicinity of $\nu_{\rm t}$ (for a more detailed analysis see Felten & Rees 1972). This simplifies the problem of calculating the Comptonization spectrum.

To find the spectrum resulting from thermal Comptonization of synchrotron radiation we use a method of Zdziarski (1986). Consider a homogeneous, isotropic sphere of Thomson optical depth $\tau_{\rm es} = n_{\rm e}\,\sigma_{\rm T}\,R$, magnetic field B and electron temperature Θ. It is assumed that given *any* internal source of monochromatic soft photons being Comptonized the resulting spectrum is a superposition of a power-law spectrum with a cut-off and a Wien spectrum. Both heuristic considerations and Monte Carlo results confirm this assumption. Then the density of photons in the spectrum can be expressed as ($x = \frac{h\nu}{m_{\rm e}c^2}$):

$$n(x) = \frac{1}{2} N_P \left(\frac{x}{\Theta}\right)^{-\alpha} \frac{e^{-x/\Theta}}{x} + \frac{1}{2} N_W \left(\frac{x}{\Theta}\right)^{3} \frac{e^{-x/\Theta}}{x} \tag{13}$$

where N_P and N_W are normalizations of the power-law and Wien spectrum components, respectively.

For $\tau_{\rm es} \sim 1$ (this is the regime we are especially interested in) the spectral index α can be determined as: $\alpha = -\ln P_t / \ln A$ where $A \approx 1 + 4\Theta + 16\Theta^2$ is the mean energy gain (per scattering) of a photon and P_t is the mean scattering probability. In the spherical geometry $P_t = 1 - 3/(8\tau_{\rm es}^3)\left[2\tau_{\rm es}^2 - 1 + e^{-2\tau_{\rm es}}(2\tau_{\rm es} + 1)\right]$.

Following Zdziarski (1986) we found, that the total intensity of the emerging radiation – Rayleigh-Jeans and Comptonization spectrum – can be approximately expressed as:

$$I = \frac{m_e c^3}{4\pi} \left\{ \frac{8\pi m_e^3 c^3 \Theta x_t^3}{3h^3} + \right.$$
$$\left. + \frac{1}{2} N_P\, \Theta \left[\Gamma\left(1-\alpha, \frac{x_t}{\Theta}\right) + \frac{\Gamma(\alpha)}{\Gamma(2\alpha+3)}\, P_t\, \Gamma\left(4, \frac{x_t}{\Theta}\right)\right]\right\} \tag{14}$$

where

$$N_P = \frac{2\varphi\, n_{RJ}(x_t)\, x_t\, \exp\left(x_t/\Theta\right)}{\left(\frac{x_t}{\Theta}\right)^{-\alpha} + \frac{\Gamma(\alpha)}{\Gamma(2\alpha+3)} P_t \left(\frac{x_t}{\Theta}\right)^3} \quad , \quad \varphi \approx \frac{1 + (2\Theta)^2}{1 + 10(2\Theta)^2} \tag{15}$$

and Γ is the incomplete Euler gamma function. Note that this formula can be easily adopted to the case of flat source geometry by recalculating the scattering probability.

Now we will pay attention to some properties of Comptonization spectra, especially those related to the characteristic of the source of synchrotron photons. Fig. 1 shows a characteristic Comptonization spectrum with parameters typical for a galactic black hole candidate. Up to the turnover frequency ν_t the source is optically thick and the Rayleigh-Jeans spectrum is formed. Above this frequency the medium becomes optically thin to absorption and Comptonization spectrum with a small Wien hump emerges as described above. The total emitted power $P_{\rm tot}$ is very sensitive to the value of magnetic field B – we found the dependence to be typically $P_{\rm tot} \sim B^{2.5\pm0.2}$. As for the power relation to the source size it is simply $P_{\rm tot} \sim R^2$ as long as $\tau_{\rm es}$ is kept fixed. This can be easily seen from the expression (14), as the intensity does not depend explicitly on R.

4. Conclusions

The formulae we have obtained for synchrotron emission coefficients and the intensity of Comptonization spectrum are both accurate and convenient in use

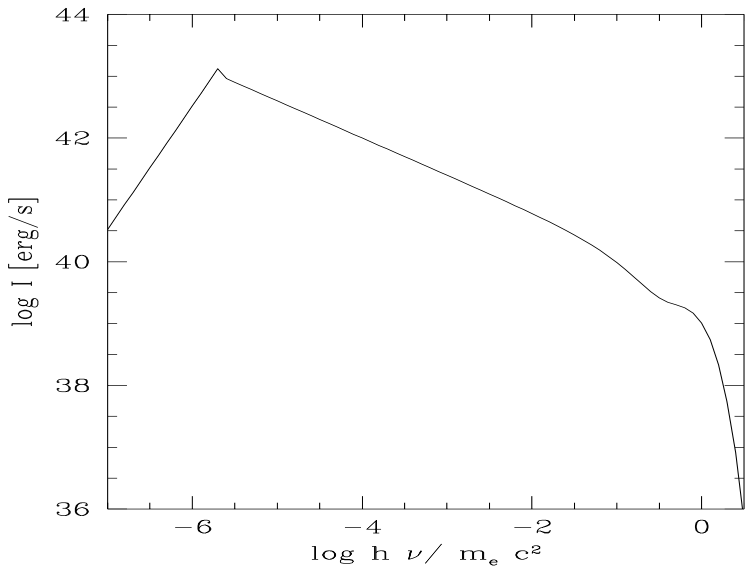

Figure 1. A typical spectrum of a source powered by thermal Comptonization of synchrotron photons. The parameters are: radius $R = 10^7$ cm, $B = 10^6$ G, $\tau_{\rm es} = 1.5$ and $\Theta = 0.2$ so that the spectral index is $\alpha = 0.6$ and the emitted power $P_{\rm tot} = 5 \cdot 10^{33}$ erg/s.

(except for the numerical calculations of the turnover frequency, however the approximations of Zdziarski et al. 1998 can be used here). They can be therefore effectively applied e.g. to solving accretion disc models with synchrotron radiation as the soft photons source, which we plan for future work.

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