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Compton Upscattering by Accretion Flows

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Abstract. We discuss Compton scattering of photons by fast moving electrons in accretion flows around compact objects. We show that the spectra emerging from such systems depend sensitively on the detailed properties and radial profiles of the emission mechanisms within the flows and may or may not be power-law functions of photon energy. We therefore conclude that power-law high-energy spectra with particular spectral indices are not distinctive signatures of accreting black holes.

1. Introduction

Compton scattering of photons by electrons with non-zero thermal and bulk velocities is the dominant radiation process in many astrophysical settings, from accreting compact objects to the early universe. The spectrum produced by Comptonization of seed photons by thermal electrons in static media is well-understood (see, e.g., Pozdnyakov, Sobol' & Sunyaev 1983). However, the spectra produced by Compton scattering by electrons in moving media are still being explored. Photon kinetic equations that are computationally tractable and correctly describe Comptonization in both the diffusion and free-streaming regimes as well as numerical algorithms for solving such photon kinetic equations in moving media and in flows near Schwarzschild black holes have been developed only recently (see, e.g., Psaltis & Lamb 1997; Zane et al. 1996)

Comptonization by moving media has been usually studied in the context of converging accretion flows, diverging wind flows, or plane-parallel radiative shocks. The formation of the radiation spectrum in such systems is often computed by solving the photon kinetic equation or the equations for its angular moments in the system frame. Here we discuss this approach. The boundary conditions are often simpler and more easily determined in the system frame, but the scattering integral is complicated because of the angular correlations between the radiation and velocity fields, which are introduced by the Lorentz transformations from the electron rest frame to the system frame. In order to simplify the photon kinetic equation and its moments, the scattering integral is usually expanded in powers of V, the electron bulk velocity divided by the speed of light, keeping only low-order terms. In most of these studies the systematic downscattering terms in the kinetic equation have been neglected, because the effect of these terms on the radiation spectrum is usually very small at photon energies $\ll m_{\rm e}V^2$, where m_e is the electron rest mass.

It has often been claimed that when low-energy photons are injected into a high-velocity converging flow, they systematically gain energy from the fast moving electrons at a rate that is proportional to the convergence of the electron velocity field (see, e.g., Blandford & Payne 1981). If this happens, the escaping radiation has a power-law spectrum (see, e.g., Payne & Blandford 1981; see also Titarchuk & Zannias 1997 and references therein) up to photon energies $\sim m_e V^2$ (Titarchuk, Mastichiadis, & Kylafis 1998; Titarchuk & Zannias 1998), with a power-law index that depends only on the mass accretion rate and the mass of the accreting compact object (Titarchuk & Zannias 1998; Shrader & Titarchuk 1998). These results were obtained by solving the radiative transfer equation expanded to various orders in V and have led some authors to argue that Comptonization by electrons with high inward radial velocities always produces a power-law X-ray spectrum extending up to hundreds of keV with a characteristic spectral index. If so, observation of such a spectrum could be used to identify accreting black holes in binary systems.

2. Is Comptonization by the Bulk Flow Possible in a Divergence-Free Velocity Field?

It has long been recognized that if the divergence of the electron bulk velocity is non-zero, the electron bulk motion causes a secular change in the energy of the photons (see, e.g., Chan & Jones 1975; Blandford & Payne 1981; Fukue et al. 1985). The photon kinetic equations derived in these works include only terms to first order in the electron bulk velocity V and predict that photons are systematically upscattered by the electron bulk motion if the electron velocity field is *converging*. However, as we show below, photons are systematically upscattered by the electron bulk motion even if the terms that are proportional to $\nabla \cdot \vec{V}$ have, on average, no effect. Indeed, the photon energy can be systematically changed, in a way that is described by terms of order V^2 or higher, even in divergence-free flows. Under some circumstances photons can be systematically upscattered by the bulk motion even if the flow is *diverging*.

The order in V of a given term in the various moments of the radiative transfer equation does not by itself determine whether the term should be retained in solving a given transport problem. This is because the terms of different order in V involve different angular moments of the radiation field, so the relative sizes of the moments must also be considered in determining which terms should be retained (see also Yin & Miller 1995).

For example, in the zeroth moment of the radiative transfer equation, which is a scalar equation for the energy density of the radiation field, the terms that are first-order in V involve the scalar product of the vector quantity $\vec{V}$ with the vector quantity defined by the radiation field, i.e., the radiation flux $\vec{H}$. In contrast, the terms that are second-order in V do not involve $\vec{H}$ but instead involve the scalar quantity defined by the radiation field, i.e., the radiation energy density J, and the radiation stress-energy tensor K^{ij}. In general, $V_iV_jK^{ij}$

is of the same size as $V^2 J$. Hence, if J is sufficiently large compared to H, the terms in the zeroth moment of the radiative transfer equation that are second-order in V can become as large as the terms that are first-order in V.

The radiation field quantities that appear in terms that involve only *odd* powers of V are all of about the same size. Similarly, the radiation field quantities that appear in terms that involve only *even* powers of V are all of about the same size, although they may be much larger or smaller than the radiation field quantities that appear in the terms that involve odd powers of V. Hence, when V is $\ll 1$, terms of order V^3 and higher may be safely neglected in the derivation of the transfer equation (see also Yin & Miller 1995).

3. Power-Law Spectra to Order V

In a spherically symmetric flow the monochromatic radiation energy density $4\pi J$ and radiation flux $4\pi H$ depend only on the radius r and the photon energy ϵ. The steady-state zeroth moment of the radiative transfer equation, expanded to first order in the radial velocity V, is (Psaltis & Lamb 1997, eq. [34])

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(H^r r^2\right) = -n_e \sigma_T V \frac{\partial}{\partial \ln \epsilon} H + \eta(r, \epsilon) \ , \tag{1}$$

where n_e is the electron density, σ_T is the Thomson scattering cross section, $\eta(r, \epsilon)$ is the emission function, which describes emission of photons within the flow, and we have also neglected the systematic downscattering and thermal Comptonization terms. In writing equation (1) we have neglected absorption of photons in order to focus on the effects of Compton scattering; although this is thermodynamically inconsistent, it does not affect significantly the shape of the high-energy spectrum produced by a cold flow.

Equation (1) is a first-order, linear partial-differential equation. Its solution is

$$H(r, \epsilon) r^2 = H_{\rm in}(\epsilon_0) r_{\rm in}^2 + \int_{r_{\rm in}}^{r} \eta(r', \epsilon') r'^2 dr' \ , \tag{2}$$

where

$$\epsilon' \equiv \epsilon \exp\left(\int_{r'}^{r} n_e(\bar{r}) \sigma_T V(\bar{r}) d\bar{r}\right) \ , \tag{3}$$

ϵ_0 is given by the same expression as ϵ' but with the lower limit of integration equal to $r_{\rm in}$ and the upper limit equal to r, and $H_{\rm in}(\epsilon_0)$ is the outward energy flux at the inner radius $r_{\rm in}$ of the flow.

When there are no sources within the flow, the spectrum of the escaping radiation is determined completely by the boundary conditions and is a power-law *if and only if* the spectrum of the radiation injected into the flow at $r_{\rm in}$ is a power-law (see also Zampieri & Lamb 1998). To see this, note that when $\eta(r, \epsilon) = 0$, equations (2) and (3) show that interaction with the flow simply shifts the input spectrum upward in the logarithm of the energy when $VH < 0$, as in the case of accretion, or downward when $VH > 0$, as in the case of an outflowing wind.

If instead there is no energy flux at the inner boundary of the flow [i.e., when $H_{\rm in}(\epsilon_0) = 0$], the emerging spectrum is determined entirely by the properties of the radiation sources within the flow. As an example, we consider a monochromatic emission function that is non-zero only at energy $\epsilon_{\rm in}$, with radial and photon-energy dependences that are separable. We also assume a power-law accretion flow with the radial velocity profile $V = -V_0 r^{\beta}$; for $V_0 > 0$ and $\beta = -1/2$ this profile describes inward free fall. Then the continuity equation for the electrons implies $n_e = n_e^0 r^{-\beta-2}$. In the limit $r \to \infty$ and for photon energies $\leq \epsilon_{\rm in} \exp(n_e^0 \sigma_T V_0 / r_{\rm in})$, the solution for the radiation energy flux is

$$H(r,\epsilon)\, r^2 = \frac{\left(n_e^0 \sigma_T V_0\right)^3}{\epsilon_{\rm in}} f(r_0') \ln\left(-4\epsilon/\epsilon_{\rm in}\right) \;, \quad \text{where} \quad r_0' = \frac{n_e^0 \sigma_T V_0}{\ln(\epsilon/\epsilon_{\rm in})} \tag{4}$$

and $f(r')$ describes the radial dependence of the emission function. Equation (4) demonstrates that the radiation flux is a decreasing power-law of the photon energy *if and only if* the radial dependence of the emission function satisfies

$$f(r) \propto \frac{1}{r^4} \exp\left(-A/r\right) \;, \tag{5}$$

where A is an arbitrary positive constant. Hence, *an emission function with any other radial dependence will produce an emerging spectrum that is not a power-law.* This argument is valid in both the diffusion and free-streaming regimes and is also independent of whether the flow converges or diverges. It shows that if only terms first-order in V need to be considered, then a power-law spectrum is very unlikely.

4. Power-Law Spectra to Order V^2

When the photon mean-free path to electron scattering is much smaller than the product of V with the characteristic length scale of the system, the terms in the radiative transfer equation that are second-order in V become at least as important as the terms that are first-order and should not be neglected (Yi & Miller 1995; Psaltis & Lamb 1997).

We have computed the spectrum of the radiation that emerges from spherically symmetric radial flows onto compact objects for a wide variety of boundary conditions and emission functions, taking into account systematic downscattering to first order in ϵ/m_e and bulk motion to order V^2. When the terms of order V^2 are included in the radiative transfer equation, its first two moments depend on the first *four* moments of the specific intensity and therefore this system cannot be solved unless two closure relations are specified. In X-ray Comptonization problems, the closure relations depend on photon energy, even though at low photon energies the photon mean-free path to electron scattering is almost energy-independent (Psaltis & Lamb 1997). Hence, instead of adopting some *a priori* closure relations in order to obtain a solution of the moment equations, we solve the full radiative transfer equation exactly, using a generalization of the method of variable Eddington factors (see Mihalas 1978). Our solutions are therefore valid whether the photon mean free path is larger or smaller than the other important lengths in the system.

Figure 1 illustrates the effects of flow properties and computational approximations on the computed spectrum of the radiation escaping from radial flows onto compact objects. The gas was assumed to be cold and falling freely onto

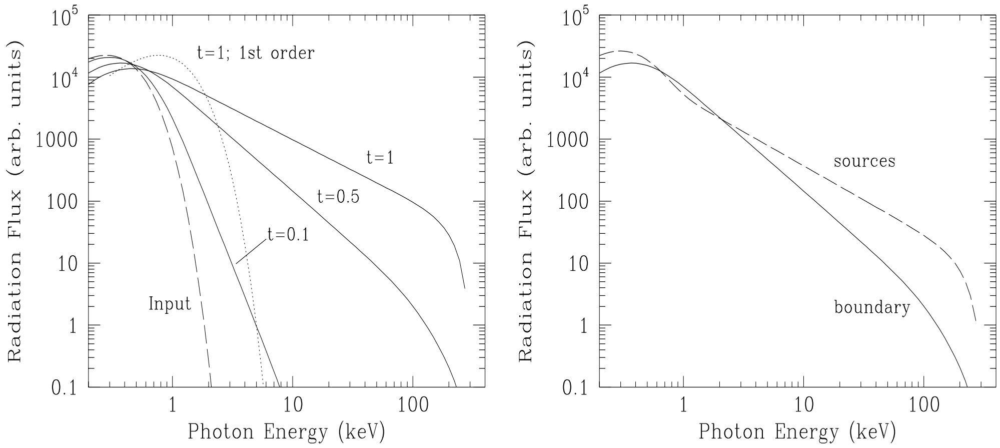

Figure 1. (a) Spectra of the radiation emerging from flows illuminated at their inner boundaries by a blackbody spectrum with a temperature of 0.1 keV (dashed line) when terms up to first-order (dotted line) and second-order (solid lines) in V are retained in the transfer equation. (b) Spectra emerging from radial flows with $t = 0.5$ when there are no internal sources of photons but the flow is illuminated at its inner boundary by 0.1 keV blackbody radiation (solid line) and when internal sources with a 0.1 keV blackbody radiation spectrum are distributed within the flow according to the square of the electron density profile but there is no illumination of the inner boundary of the flow (dotted line).

a compact object with a mass equal to ten times the mass of the sun. Absorption was assumed to be negligible and the radiative transfer equation was solved where it is accurate (i.e., where the flow velocity is $< 0.7c$). The importance of the terms in the transfer equation of order V^2 is indicated by the magnitude of the quantity

$$t \equiv -\int_{r_{\rm in}}^{\infty} n_e \sigma_T V\, dr \ , \tag{6}$$

where $r_{\rm in}$ is the radius of the inner boundary of the flow (Yi & Miller 1995; Psaltis & Lamb 1997).

The numerical results shown in Figure 1a confirm that when the effects of bulk motion are included only to first order in V, a flow that contains no radiation sources simply shifts the spectrum that is incident on the inner boundary of the flow upward in the logarithm of the photon energy (see also Zampieri & Lamb 1998). Only, when the effects of bulk motion are included to second order in V, are power-law tails generated. For a radial flow onto a black hole, t depends only on the mass accretion rate onto the black hole and the black hole mass. At first sight it may therefore appear possible that these properties of an accreting black hole can be inferred observationally from the shape of its high-energy X-ray spectrum. This is not so, however, because the spectrum produced by such a flow depends sensitively on the properties of the flow and hence is not universal.

This is illustrated by Figure 1b, which shows the spectrum produced by radial flows with the same value of t (0.5) but different radiation sources. The solid curve shows the spectrum from a flow with a free-fall density profile that has no internal sources of photons but is illuminated at its inner boundary by 0.1 keV blackbody radiation. The dashed curve shows, for the sake of illustration, the spectrum produced by a flow with a free-fall density profile if there are internal sources with a 0.1 keV blackbody radiation spectrum distributed within the flow according to the square of the electron density profile but no illumination of the inner boundary. Although we used the same radial profiles for the electron velocity and density, the power-law indices of the calculated spectra were different in the two cases considered (~ 3 and ~ 2, respectively).

5. Conclusions

The results presented here show that the high-energy X-ray spectra produced by Comptonization by high-velocity radial flows are sensitive to the properties of the internal and external sources of radiation. When only terms that are first-order in V need to be considered in the radiative transfer equation, the calculated radiation spectrum is generally not a power-law function of photon energy. When terms that are second-order in V need to be considered, the calculated radiation spectrum may be a power-law function of photon energy but the spectral index depends strongly on the properties of the sources of the radiation. Therefore, power-law high-energy spectra with a particular spectral index are not distinctive signatures of Comptonization in fast moving media, such as accretion flows onto black holes.

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