The Turbulent Solar Dynamo: State of the Art

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Abstract

After a short review of the history of the solar dynamo problem and the classic dynamo models we state what we think are the most important questions and problems that contemporary dynamo theory is facing.

1 Introduction

The Sun is an active star. Although it has shone with only relatively little variation in its luminosity since long before the dawn of mankind it is still one of the most active astronomical objects known — our own Earth is effected strongly by it: The famous "solar-wind" that sweeps through the magnetic field of the Earth owes its existence to the activity of the Sun.

One way to determine the periodicity of the solar activity is by looking at the behavior of the *sunspots* on the surface of the Sun. The sunspots follow a very regular activity cycle: In 11 years time the pattern that the sunspots form on the solar surface move from high latitudes to the equator (on both hemispheres) and then appear again at high latitudes. The *polarity* of the magnetic field of the Sun reverses every time and hence the "real" period of the solar cycle — the time it takes to get back to the same pattern and polarity — is 22 years.

The magnetic field of the Sun is detected by the Zeeman splitting of hydrogen lines in the light emerging from the relatively dark sunspots and other magnetic areas.

Sunspots are magnetic flux tubes that penetrate the photosphere of the Sun. Because such a tube is in pressure equilibrium with its surroundings the mass density inside the tube must be smaller than the mass density of the surroundings and therefore it rises to the surface in a stratified atmosphere. This is just the usual

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buoyancy also associated with balloons filled with helium in the atmosphere of the Earth. The energy flux emerging from such a tube when it comes through the atmosphere must be smaller than the energy flux from the surroundings because the magnetic field inside the tube prohibits energy transport by convection and allows only radiative energy transport; therefore the sunspots are darker than their surroundings.

Okay, — but where do these magnetic flux tubes then come from? How are they formed? It is thought that these tubes are formed in or just below the convection zone and then rise by buoyancy up through the convection zone until they encounter the photosphere and form sunspots or other magnetic structures. But the origin and the history of evolution of the tubes are not well-known at all!

The current belief is that some kind of *dynamo* process is taking place in the narrow so-called undershoot region just between the radiative dominated interior of the Sun and the overlaying convection zone. This dynamo converts kinetic energy into magnetic energy, just as a bicycle dynamo does.

2 Theoretical Background for the Dynamo

The presence of magnetic fields in many astrophysical objects raises the question of their origin. Dynamo theory suggests that magnetic fields are generated by the motions of conducting fluids. The only alternative is to assume that magnetic fields are primordial[1]. Some scientists go as far as advocating that turbulent diffusion is strongly suppressed in low-diffusion plasmas and that, consequently, astrophysical dynamos are impossible. Thus, one should consider the possibility of primordial fields. But this conclusion is based on an over -interpretation of a (theoretical and numerical) 2D-scenario. In three dimensions magnetic fields have ways of moving that are not available in two dimensions. The consequence is that turbulent diffusion of large scale structures is possible and that it is not significantly suppresed in an intermittent plasma.

But what is a dynamo? A dynamo can be defined to be a process in which some of the kinetic energy of an electrically conducting body is converted into magnetic energy. The basic principle can briefly be stated as follows: Astrophysical systems are composed of a highly electrically conducting gas made up of freely moving charged particles (electrons, protons and ions). Such a gas is called a plasma. A consequence of the high electrical conductivity is that the magnetic fields are "frozen" into the plasma (as first discussed by Alfven). Thus, plasma can move freely along field lines, but in motion perpendicular to them, either the field lines are dragged with the plasma or the field lines push the plasma.

It has been found that "turbulent" motions of the plasma can amplify small seed magnetic fields and generate much larger magnetic fields. More particularly these motions could give rise to fields that periodically oscillate and this could possibly explain the solar activity 22 year cycle.

In this framework the Sun presents us with an oscillating dynamo that can

[1]Primordial fields can be defined as fields that have been around too long to need explaining.

be observed in great detail. The turbulence in the convection zone creates the differential rotation through transport of angular momentum. Differential rotation tends to pull out a purely poloidal field and create toroidal flux (toroidal $\sim$ azimutal, see the next section). When this azimuthal field becomes sufficiently strong the spiral lines of force are shaped like Ω-loops that arise to the surface because of magnetic buoyancy. In the process of rising they are rotated by Coriolis force, so that a (new) poloidal field is generated in the opposite direction to the original one. The role of the turbulent transport is to spread the new poloidal field. This scenario leads to an oscillation between a poloidal and a toroidal magnetic field.

Our limited understanding of convection and turbulence is the main obstacle in making more qualitative models of the above scenario. Turbulence shreds, amplifies and transports the field. A basic property of a turbulent plasma is the tendency to become extremely intermittent; vorticity concentrates into thin vortex tubes and the magnetic field concentrates into thin flux tubes. The turbulent diffusion is also central to classical dynamo arguments. Locally, the magnetic energy density can become comparable to the kinetic energy density long before an estimate of the amount of magnetic energy based on a diffuse field would. In this way, we are reconsidering classical concepts such as turbulent diffusion of magnetic fields. It is the detailed properties of the turbulence that determine what the dynamo will look like.

2.1 *Cowlings theorem*

Many years ago Cowling discussed the dynamo problem and sharpened it by showing that "*a steady axisymmetric magnetic field cannot be maintained by fluid motions*". It may be established as follows (see eg. [5] or [6]).

We can write a steady axisymmetric field as the sum of a toroidal component B_ϕ and a poloidal component B_p. The toroidal component is an azimuthal component and the poloidal component represents the sum of the radial and axial components in cylindrical coordinates.

$$\mathbf{B} = B_\phi \hat{\mathbf{e}}_\phi + \mathbf{B}_p \tag{1}$$

Because of the axisymmetry the magnetic configuration in all meridional planes is the same and must consist of closed field lines (The meridional planes are planes through the axis of the symmetry). In each of these meridional planes there must therefore exist at least one point with $\mathbf{B}_p = 0$. Hence, the field is purely azimuthal ($\mathbf{B}_\phi \neq 0$). Now Ohm's Law in the form $\mathbf{j}/\sigma = \mathbf{E} + \mathbf{v}\times\mathbf{B}$ may be integrated around the azimuthal neutral line (c) (ie. Figure 1) to give

$$\oint_c \mathbf{j}/\sigma \cdot \mathbf{ds} = \oint_c \mathbf{E}\cdot\mathbf{ds} + \oint_c \mathbf{v}\times\mathbf{B}\cdot\mathbf{ds} \tag{2}$$

or

$$\oint_c j_\phi ds/\sigma = \int_s \nabla \times \mathbf{E}\cdot\mathbf{dS} + \oint_c \mathbf{v}\times\mathbf{B}\cdot\mathbf{ds}, \tag{3}$$

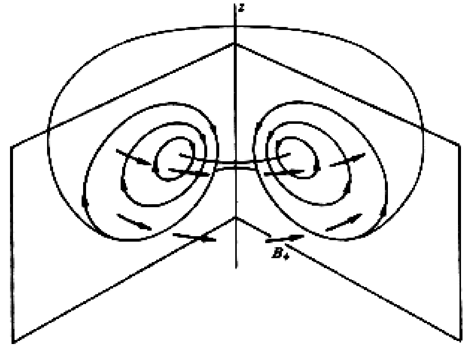

Figure 1: A sketch of an axi-symmetric field configuration, made up of an azimuthal field,$\mathbf{B}_\phi$shown by the heavy arrows, and a dipole meridional field, indicated by the closed lines of force circling the neutral point in the meridional planes

where the element of length **ds** is in the ϕ-direction. Stoke's theorem has also been used to transform the first term on the right-hand side.

We assumed that the magnetic field is steady. That means $\nabla \times \mathbf{E}$ vanishes due to Faraday's Law. At neutral points $\mathbf{B}$ is parallel to the path elements $\mathbf{ds}$ so that the triple scalar product $\mathbf{v}\times\mathbf{B}\cdot\mathbf{ds}$ vanishes and the integral of Ohm's Law reduces to $\oint_c j_\phi ds = 0$. But since the current j_ϕ is non vanishing in a neighborhood of a neutral point this cannot be satisfied. The conclusion is that a steady field cannot be axisymmetric. We had to emphasize that Cowling's argument holds only for exact axisymmetry. Departures from axisymmetry may allow a dynamo to work.

2.2 *Dynamo Theories*

In dynamo theories a magnetic field is maintained by currents induced in a plasma by its motion across lines of force. In this point of view we consider the basic physic of the interaction between plasma and magnetic field. To see this we start by writing down the relevant *magneto-hydrodynamics* (MHD) equations:

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \tag{4}$$

$$\nabla \times \mathbf{B}/\mu = \mathbf{j}, \tag{5}$$

$$\mathbf{E} = \mathbf{j}/\sigma - \mathbf{u} \times \mathbf{B}, \tag{6}$$

where $\mathbf{B}$ is the magnetic field, $\mathbf{E}$ is the electric field, $\mathbf{u}$ is the velocity, σ the electric conductivity, μ the magnetic permeability and $\mathbf{j}$ the electric current respectively. By using equations (5) and (6) we rewrite the induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \tag{7}$$

where $\eta = 1/(\mu\sigma)$ is the magnetic diffusivity (for $\mathbf{u} = 0$ equation (7) is a diffusion equation). We have also used the identity $\nabla \times (\nabla \times \mathbf{B}) = \nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B}$ and the condition $\nabla \cdot \mathbf{B} = 0$ which expresses the absence of magnetic monopoles. The induction equation is the basic equation for magnetic behaviour in MHD; it determines **B** once **u** is known. The basic physic is that the plasma velocity (**u**) and magnetic field are the primary variables, determined by the induction equation, while the resulting current (**j**) and electric field (**E**) are secondary and may be deduced from equations (5) and (6) if required.

The equations we have set up in this subsection imply some relationships between the different types of energy such as heat, electrical energy and magnetic energy. First of all the divergence of the Poynting flux $\mathbf{E} \times \mathbf{B}$ may be transformed, using equations (4) and (5), to

$$-\frac{1}{\mu}\nabla \cdot (\mathbf{E} \times \mathbf{B}) = \mathbf{E} \cdot \mathbf{j} + \frac{\partial (B^2/2\mu)}{\partial t} \tag{8}$$

Its physical interpretation is that an inflow of electromagnetic energy $\mathbf{E} \times \mathbf{B}$ produces electrical energy $\mathbf{E} \cdot \mathbf{j}$ for the plasma and a rise in magnetic energy $B^2/2\mu$. In turn, the electrical energy given to the plasma by the electromagnetic field may be rewritten, after substituting for **E** from equation (6), as

$$\mathbf{E} \cdot \mathbf{j} = \frac{j^2}{\sigma} + \mathbf{u} \cdot \mathbf{j} \times \mathbf{B} \tag{9}$$

This means that the electrical energy appears as heat by ohmic dissipation (turns magnetic energy into kinetic energy or conversely) and work done by $\mathbf{j} \times \mathbf{B}$ force (turns magnetic energy into heat).

The first term on the right hand side of the induction equation describes the inducing effect of the motion of the fluid upon the magnetic field. The second term describes the "ohmic" decay of the field due to the finite electrical resistance. We may compare the two terms in equation (7) (to the order of their magnitudes) if we replace the divergence vectors by their absolute magnitudes and the curl operator by $1/l$, where l is the scale of field variation in space. We obtain

$$R_m = \frac{ul}{\eta}, \tag{10}$$

as the ratio of the induction term over the decay term; R_m is called the magnetic Reynolds number. Thus, this number measures the ratio of advective to resistive effects for structures of size l advected by velocities of the order of u. In solar physics one often speaks of "high conductivity scenarios". The precise meaning of this is $R_m \gg 1$ or that the time scale of ohmic decay ($\tau = l^2/\eta$) is very much longer than l/u which is the "advection time scale".

A more specific and interesting topic is diffusion of a three dimensional magnetic field by three dimensional turbulence and the behaviour of turbulent transport in systems with very high magnetic Reynolds numbers.

Three dimensional turbulence allows one particularly important transport effect which is not accessible with strictly two-dimensional motion: Even if the three dimensional motion is perfectly incompressible ($\nabla \cdot \mathbf{u} = 0$) there is in general a divergence or a convergence of the flow along the magnetic field, which must be

Figure 2: The effect of increased numerical resolution on 3D-turbulent magnetic diffusion. The left half was computed at a resolution of 64^3 while the right half was computed at a resolution of 128^3.

compensated for by a corresponding convergence or divergence of the velocity in the plane perpendicular to the magnetic field. Thus, one may consider, over the entire volume, the "perpendicular convergence"

$$-\nabla \cdot \mathbf{u}_\perp = -\nabla \cdot (\mathbf{u} - \mathbf{B}(\mathbf{u} \cdot \mathbf{B})/B^2) \tag{11}$$

Apart from diffusive effects the quantity gives the exponential rate of growth of the local magnetic field amplitude, following the field line motion;

$$\mathbf{D}_\perp ln|\mathbf{B}|/Dt = -\nabla \cdot \mathbf{u}_\perp \tag{12}$$

Local and global maxima of this function are the places where the magnetic field grows most rapidly with time. If the flow is sufficiently persistent, so that such local maxima exist for times of the order of a *turn-over time* the local field strength has time to grow exponentially, and thus form strongly localized concentrations.

Recently, numerical experiments [2] elaborated the diffusion of a 3D-magnetic field by 3D-turbulence. In this case we can see (Figure 2) that even for high numerical resolution [2], diffusion of an equipartition ($E_{magn} \approx E_{kin}$) strength magnetic field is not suppressed.

The issue of a possible suppression of turbulent transport at high Reynolds numbers is of importance for attempts to understand the solar dynamo. First of all because a significant turbulent diffusion would seem to be necessary to destroy the dynamo field from one half-cycle to the next. On the other hand transport effects such as the "alpha effect" could be suppressed in a similar fashion. But the generation of new field requires the existence of something like an alpha effect. Thus, we could face a situation where both the growth and decay time scales would be much too long to explain the solar dynamo.

What could be done about this interesting point of turbulent dynamo theory? Eg. we could simply study the growth rate of a turbulent dynamo and see eg. whether

[2] we decrease the size of the dissipating structures which corresponds to high magnetic Reynolds number

of not a high R_m situation is at all compatible with having a "fast dynamo", i.e. one that grows with a time scale of the order of the turn-over time. We obtain

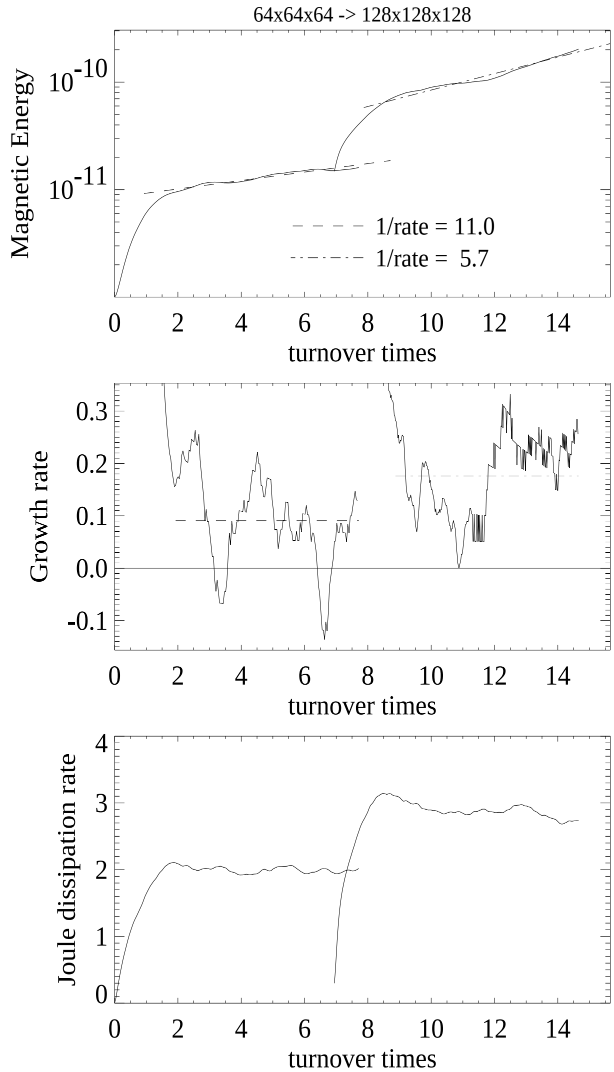

Figure 3: The effect of increased numerical resolution on turbulent dynamo action, in an experiment with isotropic turbulence. The first part of the experiment is done with a numerical resolution of 64^3, the second part with 128^3. The dashed and dashed-dotted lines are fits to the exponential growth of magnetic energy. The mid panel shows the instantaneous and average growth rates, and the bottom panels shows the rate of energy loss due to Joule dissipation (Nordlund et al. 1994)

some interesting results from such an experiment [2] in figure 3.

- The growth rate increases with higher numerical resolution (higher R_m).
- The growth rate increases even if the Joule dissipation time E_{magn}/Q_{joule} decreases. This is remarkable because the Joule dissipation time is shorter to begin with than inverse of the magnetic rate.

These results indicate that there is a "detailed balance" between the work done by the flow on the magnetic field and the Joule dissipation. Since the size of the small scale magnetic flux tubes remains the same (on the average) over time these two contributions (magnetic field work and resistive diffusion of plasma through magnetic field) must be in close balance.

2.3 *Turbulence and Flux Tubes*

Additional factors that governs the turbulent transport of the magnetic field are vorticity, magnetic flux tubes and buoyancy. More specific, flux and vortex tubes are formed in the solar convection zone.

Vorticity is defined as the curl of the velocity $\omega = \nabla \times \mathbf{u}$ and is thus a quantitative description of the rotation of the velocity field. A vortex tube has a spiraling of fluid towards the center of the tube and out along the tube [1] and thus vorticity meassures the angular momentum of the fluid around the rotational axis of the vortex tube.

Magnetic flux tubes arise as concentrations of the magnetic field on a fast time scale in a general 3D magnetic field. There is an intimate connection between the strong magnetic field and the turbulent motions inside the tube (Figure 4; in this figure we can see that there is a strong coupling between vortical and magnetic phenomena). In Figure 4 a vortex tube (light surface) is closely overlapped by a flux tube (dark surface) with magnetic fieldlines running pretty much in parallel. At the end of the flux tube (center and right) the fieldlines spread out, connecting to another coherent structure (a flux tube). The intermediate field is relatively weak and diffuse.

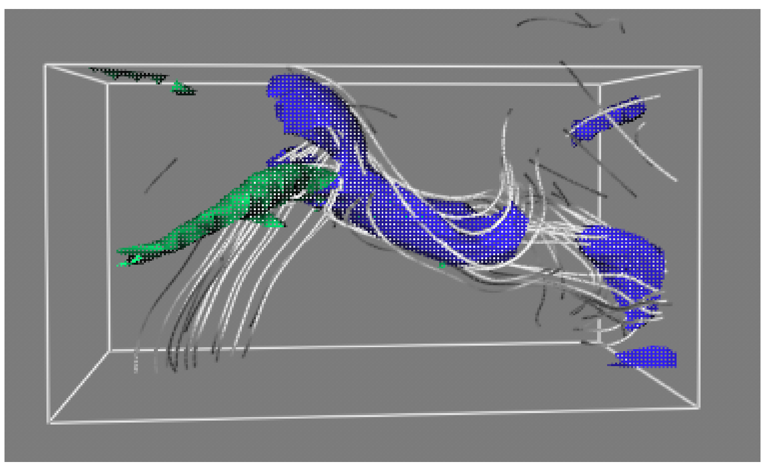

Figure 4: Visualization of the results of a numerical simulation of the magneto-hydrodynamical equations.

The fundamental paper by Parker [4] suggests that when a flux tube is formed it rises by magnetic buoyancy and produces a sunspot where it breaks the photospheric surface. When such a rising tube creates a *pair* of spots — when it is "Ω"-shaped — flux erupt from the region between these newly formed spots in the form of a bipolar *loop* structure.

Nevertheless, such structure is not a necessity. Often the flux of one polarity is more concentrated than that of the other so that only a single "unipolar" visible spot is formed.

The smallest magnetic phenomena on the Sun which can be distinguished in white light are the "pores". Pores can be almost as dark as sunspots but have no penumbra and are generally much smaller. Almost invisible in white light but very similar in their magnetic structure are the magnetic "knots". The magnetic knots represent the intersection of magnetic flux tubes with the surface of the sun with fields of 1100 - 1400 G (Beckers and Scroter 1968). An important clue to the cause of the attraction is that it extends only to magnetic flux tubes nearly arriving at the surface. Once there no new flux tubes arriving at the surface the knots lose their mutual attraction and begin to move apart (as one would expect them to do).

The importance of the magnetic buoyancy effect on magnetic flux tubes can be estimated from the size of the density deficit

$$\frac{\rho_e - \rho_i}{\rho_e} = \frac{\mathbf{B}_i^2 m}{2\mu\rho_e\kappa_B T}. \tag{13}$$

At a distance 20 Mm below the photosphere the magnetic buoyancy is small $(\rho_e - \rho_i)/\rho_e \approx 10^{-5}$. On the other hand at only 1 Mm below the photosphere one may take $(\rho_e - \rho_i)/\rho_e \approx 0.004$. Therefore the buoyancy force is much stronger in the upper part of the convection zone. Long-term flux storage seems possible at the bottom of the convection zone or in the convectively stable layer immediately below (Spiegel and Weiss, 1980). It is therefore now generally believed that much of the dynamo operates near this boundary between the radiative core and the convection zone of the sun. Another interesting task is to explore the dynamical effects of

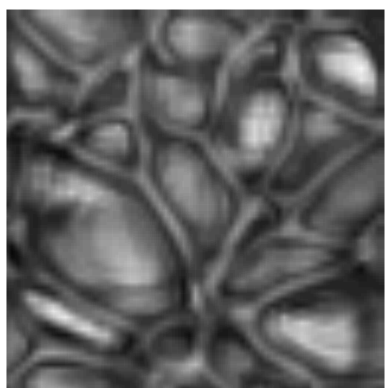

Figure 5: The top slice of a deep solar model is shown: The vertical velocity component u_x (black is minimum and white is maximum). A granulation pattern is clearly seen to occur as a consequence of convection.

external turbulence on an isolated tube of flux. The flux tubes observed in the Sun are confined by the converging flows on the surface to the convective downdrafts between granules and supergranules (see Figure 5 for a model of this). Thus they are subjected to downward drag by the external turbulence (the turbulent external fluid is generally streaming downward along the tube). The effect is a downward pumping of the fluid within the tube partially evacuating the interior and compressing the magnetic field.

An additional effect is due to the association of flux tubes with cool spots at the surface. This fills the flux tubes with entropy deficient fluid. A consequence of this is the reducing of the buoyancy otherwise associated with a concentrated magnetic field.

3 Dynamo Action

The dynamo problem may be divided into some interesting subdomains. We shall make a brief description of kinematic, mean field and magnetohydrodynamic dynamos.

3.1 *Kinematic dynamo*

In this dynamo model we neglect all the feedback of the magnetic field on the fluid flow topology. In paragraph (2.2) we formed and described some basic properties of the induction equation. If we have a prescribed velocity field which goes into this equation we get a magnetic field. We can simply study the growth rate of the magnetic field to obtain (or not to obtain) a dynamo process. If the magnetic field has a positive growth rate then we have dynamo action. One says the dynamo is "fast" when the growth rate remains finite in the limit of zero electric resistivity η (that means we refer in regions with large magnetic Reynolds numbers). On the other hand the dynamo is "slow" if the growth rate approaches zero.

3.2 *Mean Field Dynamo*

In this section we seek a solution to the dynamo problem in terms of a mean magnetic field. Thus, it would be nice to keep in mind Parker's suggestion: the net effect of averaging many small-scale convective motions would be to produce the large scale electric field and so allow regeneration of the poloidal magnetic field. Hence we write

$$\mathbf{B} = <\mathbf{B}> + \mathbf{b}, \tag{14}$$

where $<\mathbf{B}>$ is an average over longitude or more generally as an ensemble average. The field $\mathbf{b}$ is the fluctuating part of $\mathbf{B}$ and obeys $<\mathbf{b}>= 0$. We also write the velocity as a sum of a global mean and a fluctuating part.

Substitution of equation (14) (and the corresponding equation for the velocity) into the induction equation (7) and separation of the mean and fluctuating parts yields two partial differential equations for the mean and the fluctuating field respectively. The mean field equation becomes:

$$\frac{\partial <\mathbf{B}>}{\partial t} = \nabla \times (<\mathbf{v}> \times <\mathbf{B}> + Q - \eta \nabla \times <\mathbf{B}>), \tag{15}$$

where

$$Q =< \mathbf{u} \times \mathbf{b} > . \qquad (16)$$

The crucial quantity is the mean electric field Q. If we knew this quantity we could solve (15) for $< \mathbf{B} >$. In general one could imagine a linear relationship between Q and $< \mathbf{B} >$ (also between $\mathbf{b}$ and $< \mathbf{B} >$). In the case where $\mathbf{b}$ represents "weakly" isotropic turbulence we write the latter relationship as an expansion:

$$Q = \alpha < \mathbf{B} > -\beta \times \nabla \times < \mathbf{B} >, \qquad (17)$$

where α and β are constants. Substitution of (16) into (12) yields the mean field induction equation:

$$\frac{\partial < \mathbf{B} >}{\partial t} = \nabla \times (< \mathbf{v} > \times < \mathbf{B} > +\alpha < \mathbf{B} > -(\eta + \beta)\nabla \times < \mathbf{B} >) \qquad (18)$$

The term $\alpha < \mathbf{B} >$ represents the mean electromotive force due to turbulent flow ("alpha effect"). This term ensures that the solution $< \mathbf{B} >$ is not subject to the antidynamo theorem mentioned in the preceding sections. On the other hand side term β describes the diffusion of the magnetic field due to the turbulent flow.

Equation (18) represents the so-called "$\alpha - \omega$" dynamo because the term $< \mathbf{v} > \times < \mathbf{B} >$ generates toroidal field from a poloidal field due to the differential rotation of the sun and $\alpha < \mathbf{B} >$ generates poloidal field from a toroidal one due to the turbulent flow. The mean field induction equation has been the basis for much work on the solar dynamo and even for the terrestrial and for stellar dynamos. But although some results have been obtained it is neccesary to remember the assumptions and approximations involved here: There is not any physics that predicts the value of the α. In fact, to reproduce the solar cycle length (22 years) one has to use a very small α of the order of 10^{-2}, but the "observed" α is much larger (in the order of unity)! Additionally, why should it be relevant to make great ado about calculating a global mean field when the physics of the dynamo effect is maybe taking place at a smaller scale?

3.3 *Magnetohydrodynamic Dynamo*

The more direct approach to model the solar dynamo would be by simulating the basic physical conditions that we know the Sun is subjected to. We have to take into account the feedback of the magnetic field on the fluid motions and we have to examine the detailed dynamical behavior of the magnetic field and the turbulent motions of the plasma.

The basic equations for the fully developed case of MHD dynamo are conservation of mass, momentum and energy and the induction equation, Ampere's Law and the general Ohm's Law in the form:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho \mathbf{u}, \qquad (19)$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} = -\nabla \cdot (\rho \mathbf{u}\mathbf{u} - \tau) - \nabla P + \rho \mathbf{g} + \mathbf{j} \times \mathbf{B} - 2\rho\omega \times \mathbf{u}, \quad (20)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \quad (21)$$

$$\nabla \times \mathbf{B} = \mathbf{j}, \quad (22)$$

$$\mathbf{E} = \eta \mathbf{j} - \mathbf{u} \times \mathbf{B}, \quad (23)$$

$$\frac{\partial e}{\partial t} = -\nabla \cdot (e\mathbf{u} + \mathbf{F}_{diff}) - P\nabla \cdot \mathbf{u} + Q_{rad} + Q_{visc} + Q_{Joule}, \quad (24)$$

where ρ are the density, $\mathbf{u}$ velocity field, $\mathbf{B}$ magnetic field, $\mathbf{E}$ electric field, P gas pressure, η electric resistivity, $\mathbf{j}$ electric current, τ viscous stress tensor, e internal energy, F_{diff} diffusive energy flux, Q_{rad} radiative energy transfer, Q_{visc} viscous and Q_{Joule} Joule dissipation respectively.

The first equation describes the conservation of mass for a in general compressible fluid. The second equation describes the conservation of momentum $\rho\mathbf{u}$ (per unit mass). On the right-hand side of this equation we include the gravitational force, the centrifugational force, the Lorentz force and the Coriolis force. The symbol $\mathbf{uu}$ is the tensor with the elements $u_i u_k$ and ω is the angular velocity (vector). The other three expressions is just the Maxwell equations allready discussed.

Finally we need an equation to specify the transport of energy. Equation (24) specify the rate of change of internal thermal energy density e. On the right-hand side of this equation we have the divergence of the energy flux term that transport heat to a volume element and work and dissipation terms which change other energy forms into heat in the volume. Thus, the rate of internal thermal energy density must be equal to:

$$\frac{\partial e}{\partial t} = -\nabla \cdot \mathbf{F} + W_{gas} + Q_{rad} + Q_{visc} + Q_{Joule}, \quad (25)$$

where

$$-\nabla \cdot \mathbf{F} = -\nabla \cdot (e\mathbf{u} + P_{magn}\mathbf{u}) + u \cdot \nabla P - P\nabla \cdot \mathbf{u}, \quad (26)$$

$$W_{gas} = -\mathbf{u} \cdot \nabla P. \quad (27)$$

This equation completes the set of the MHD equtions.

The modeling of the dynamo process by using the full MHD equations can only be done rightgeously on super computers. Many numerical experiments have been carried out on massive parallel super computers like the *Connection Machine* at UNI-C (eg.[2]). The programming code is, in the case of the Connection Machine, a specially developed CM-Fortran code (by [3]). All the figures in this paper have been constructed from data that have resulted from different versions of this code.

4 State of the art

Today we have reached a point where we by using fast computers (both graphically and computationally) can gain knowledge about such complicated processes as the dynamo process by just *looking* at the physics that are going on in eg. the Sun. We can construct a realistic model of the Sun (or of an accretion disc) using the most basic equations that are describing this system: The magneto-hydrodynamic equations together with an equation of state. We then run this model on a fast supercomputer and get a lot of data which we can visualize on the best graphics workstations.

This procedure enables us to gain considerable understanding about the physical processes that are involved in eg. the dynamo process and that is — after all — what we are really seeking: Understanding.

In the rest of this section we shall try to outline what it is that we do not understand and try to give some hints for the direction in which to find the answers.

A simple question that nevertheless does not have a simple answer is the following

- Q: How does the reversal of the sun's polar field work.

The standard answer would probably be this:

- A: *Great unipolar magnetic field exists like spots on the surface of the sun. These fields drift — like sunspots depending on their polarity — either towards the poles or towards the equator. Those that drifts to the poles will gradually cancel the polar field and eventually reverse it.*

The *truth* however is that we actually don't know it. It is possible to pin the question further down:

- Q: Why do fields drift like they do? Because of magnetic buoyancy one should think that when a field (a flux tube) rises through the atmosphere it would just keep on rising unless it was somehow anchored to the 'global' toroidal field at the bottom of the convection zone. On the other hand - the fields do drift and over large areas too, so how can they be anchored?

- A: *Maybe the flux tubes act like straws in a river: Once the roots have rotten away the straw floats freely just as if it was standing with some tilt? Maybe when a strong field starts rising from the bottom of the convection zone somekind af instability occurs and this field breaks 'loose' and form an independent loop structure that can float freely.*

- Q: How do the flux tubes actually rise? And when they rise, how can such a strong toroidal field exist at the bottom of the convection zone?

- A: *One might think that the coupling between descending convective elements (plumes) and the magnetic flux tubes that finds themselves near the photosphere and thus have low entropy because of surface cooling, would constantly add low entropic tubes to the relatively strong fields at the bottom of the convection zone. Thus the field would be stable against buoyancy.*

These questions are very relevant to the understanding of the solar dynamo process and many people are trying to answer them — among other things — by performing numerical simulations of very high accuracy eg. [2], [1].

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